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Commentary

Comments on "Separation of Q_i and Q_s from passive data at Mt. Vesuvius: A reappraisal of the seismic attenuation estimates" by E. Del Pezzo et al. (2006)

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ABSTRACT

In this comment we point out two approximations considered by Del Pezzo et al. [Del Pezzo, E., Bianco, F., Zaccarelli, L., 2006. Separation of Q_i and Q_s from passive data at Mt. Vesuvius: a reappraisal of the seismic attenuation estimates, Phys. Earth Planet. Int. 159,202–212] for estimating intrinsic absorption and scattering attenuation in Vesuvius volcano region. The first one is the theoretical model used for calculating the synthetic S-wave envelopes for the case of multiple scattering. The second one is based on the stacking of coda envelopes. We consider that both assumptions have no general validity for this kind of analysis and thus may have biased their results.

1. Opening remarks

There are two approximate solutions of the radiative transfer (or Boltzmann) equation in the literature for the computation of the scattered seismic wave energy density for multiple isotropic scattering (Zeng, 1991; Paasschens, 1997). On the basis of these theoretical models, which are numerically very convenient, alternative methods to the well-known multiple lapse time window analysis (MLTWA; Hoshiba et al., 1991; Fehler et al., 1992) have been proposed to estimate intrinsic and scattering attenuation in the Earth. On the one hand, Sens-Schönfelder and Wegler (2006) used a joint inversion of S-coda envelopes for source and site parameters, as well as for medium parameters assuming the theoretical model of Paasschens (1997), which is shown to be very accurate. On the other hand, Matsunami and Nakamura (2004) and Del Pezzo et al. (2006) used a method based on a fitting process of the theoretical S-coda model by Zeng (1991) to an average S-coda envelope. However, we have serious concerns about the accuracy of the attenuation parameters reported by Del Pezzo et al. (2006) in Vesuvius volcano region. In this work we point out two approximations that may have biased their results. The first one is the use of the hybrid single-scattering-diffusion approximate solution of the 3D radiative transfer equation (Zeng, 1991) for calculating the synthetic S-wave envelopes for the case of multiple isotropic scattering. The second one is the assumption that the stacked coda envelopes represent the average of absorption and scattering coefficients of the region.

2. Point 1: comparison of the exact and two approximate solutions of the 3D radiative transfer equation

Multiple scattering processes may be modelled by using the radiative transfer theory for the energy density (Ishimaru, 1978). The radiative transfer (or Boltzmann) equation can be solved exactly in the Fourier space for one, two, three and four dimensions. Zeng et al. (1991) obtained an exact compact integral solution for the real three-dimensional space. They wrote the solution as a summation of three terms: the first one corresponds to the ballistic peak

$$P_0(r,t) = \frac{1}{4\pi r^2}\delta(r - vt)\exp\left(\frac{-vt}{l}\right)\exp\left(\frac{-vt}{l_a}\right), \tag{1}$$

the second one corresponds to the contribution due to single scattering

$$P_1(r,t) = \frac{1}{4\pi lvtr}\exp\left(\frac{-vt}{l}\right)\exp\left(\frac{-vt}{l_a}\right)\ln\left(\frac{vt+r}{vt-r}\right), \tag{2}$$

and the last term accounts for multiple scattering. The contribution coming from multiple scattering is usually written as the summation of two terms. In this way the corresponding integrals may be evaluated by means of numerical integration techniques. The first term corresponds to double scattering:

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$$P_2(r,t) = \frac{1}{16\pi l^2}\exp\left(-\frac{vt}{l}\right)\exp\left(-\frac{vt}{l_a}\right)\left[\frac{\pi^2}{vt} - \frac{3}{r}\int_0^{r/vt}\left(\ln\left(\frac{1+\alpha}{1-\alpha}\right)\right)^2 \mathrm{d}\alpha\right], \quad (3)$$

and the other terms correspond to multiple scattering (excluding double scattering):

$$\sum_{N\geq 3} P_N(r,t) = \int_{-\infty}^{+\infty}\frac{\exp(\mathrm{i}\Omega t)}{2\pi}\mathrm{d}\Omega \int_0^{\infty}\frac{(1/lk)^3[\arctan(k/(1/l+1/l_a+\mathrm{i}\Omega/v))]^4\sin(kr)}{2\pi^2 vr\left[1-(1/lk)\arctan(k/(1/l+1/l_a+\mathrm{i}\Omega/v))\right]}\mathrm{d}k, \quad (4)$$

where v is the wave velocity, l is the mean free path for scattering, and l_a is the absorption length.

However, Eq. (4) may only be computed by using numerical algorithms for two-dimensional integration. It is possible to use the fast Fourier transform (FFT) algorithm (Bouchon, 1979) to evaluate the integral, but the FFT may not be an accurate algorithm when the integrand is highly oscillatory. A powerful and accurate numerical tool that will be used in this work is a two-dimensional adaptative cubature algorithm called Cubpack (Cools and Haegemans, 2003).

In order to simplify the computations, it is possible to develop two approximations of the exact solution in three dimensions: the hybrid single-scattering-diffusion solution of Zeng (1991), and the algebraic expression of Paasschens (1997). We will not consider now approximations such as diffusion, since we are interested only in analytical expressions with a wide range of validity for the parameters l and l_a.

The first approximation (Zeng, 1991) was derived by combining the single scattering and the diffusion models for short and long lapse times, respectively, and it has been used by several authors in practical analysis for synthesizing the theoretical energy density (Bianco et al., 2002 in the southern Apennine zone of Italy; Goutbeek et al., 2004 in southern Netherlands; Matsunami and Nakamura, 2004 in southwestern Japan; Bianco et al., 2005 in the northeastern Italian peninsula; and Del Pezzo et al., 2006 at Vesuvius volcano). It has the form:

$$P(r,t) \simeq E_0\exp\left(-\frac{vt}{l}\right)\exp\left(-\frac{vt}{l_a}\right)\left[\frac{\delta(t-r/v))}{4\pi vr^2} + \frac{1}{l}\frac{\Theta(t-r/v)}{4\pi vrt}\ln\frac{1+r/vt}{1-r/vt}\right] + , \quad (5)$$
$$+v\Theta\left(\frac{t-r}{v}\right)\left(\frac{3}{4\pi lvt}\right)^{3/2}\exp\left(-\frac{3r^2}{4lvt}-\frac{vt}{l_a}\right)$$

where $c = E_0[1-(1+vt/l)\exp(-vt/l)]/\left(4/\sqrt{\pi}\int_0^{\sqrt{3vt/2l}}\alpha^2\exp(-\alpha^2)\,\mathrm{d}\alpha\right)$ and $\Theta(x)$ is the step function, which is zero for $x<0$ and 1 for $x>0$.

The second approximate solution of the radiative transfer equation in 3D (Paasschens, 1997) uses the interpolation between exact solutions of (1) for the 2D and 4D cases and it was used by Abubakirov (2005) in a MLTWA for the Kamchatka lithosphere. Sens-Schönfelder and Wegler (2006) also used the Paasschens' (1997) formulation to obtain source spectra and seismic moments. The solution has the form:

$$P(r,t) \simeq \frac{1}{4\pi r^2}\delta(r-vt)\exp\left(-\frac{vt}{l}\right)\exp\left(-\frac{vt}{l_a}\right) +$$
$$\frac{(1-r^2/v^2t^2)^{1/8}}{(4\pi lvt/3)^{3/2}}\exp\left(-\frac{vt}{l}\right)\exp\left(-\frac{vt}{l_a}\right)G\left(\frac{vt}{l}\left[1-\frac{r^2}{v^2t^2}\right]^{3/4}\right)\Theta(vt-r), \quad (6)$$
$$G(x) = 8(3x)^{-3/2}\sum_{N=1}^{\infty}\frac{\Gamma((3/4)N+(3/2))}{\Gamma((3/4)N)}\frac{x^N}{N!}$$

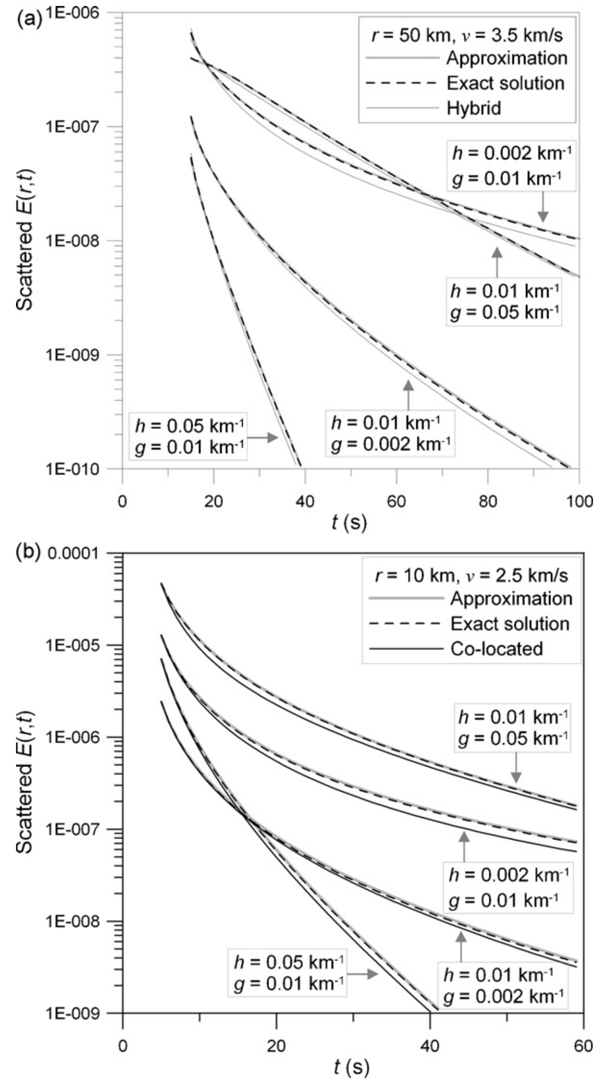

Fig. 1. Scattered wave energy as a function of time for (a) hypocentral distance $r = 50$ km and S-wave velocity $v = 3.5$ km/s; and (b) hypocentral distance $r = 50$ km and S-wave velocity $v = 3.5$ km/s. Several combinations of the absorption (h) and scattering (g) coefficients are considered. The grey, solid curves represent the approximation of Paasschens (1997) whereas the short dashed lines show the exact solution of Zeng et al. (1991) integral equation. The black, solid lines represent the hybrid single-scattering-diffusion solution by Zeng (1991).

where it is possible to approximate $G(x)$ within a few percent of error as $G(x) \simeq \exp(x)\sqrt{1+2.026/x}$.

In Fig. 1 we show a comparison of the scattered wave energy obtained by means of the numerical integration of Zeng et al. (1991) equation (expressions (1)–(4), exact solution) with the approximate hybrid single-scattering-diffusion solution of Zeng (1991) (Eq. (5)) and the analytical approximation of Paasschens (1997) (Eq. (6)), for typical combinations of the absorption ($h = 1/l_a$) and scattering ($g = 1/l$) coefficients. In general, the hybrid single-scattering-diffusion solution presents differences with the exact solution that depend on the lapse time and the values of g and h considered. For the typical example shown in Fig. 1(a) (hypocentral distance $r = 50$ km and S-wave velocity $v = 3.5$ km/s), the maximum difference between the hybrid single-scattering-diffusion approximation and the exact solution is of the order of 23%. Moreover, the difference is not constant with lapse time so, for some combinations of the attenuation parameters, r and v, the shape of the synthetic envelope may change too. As for the Paasschens (1997) expression,

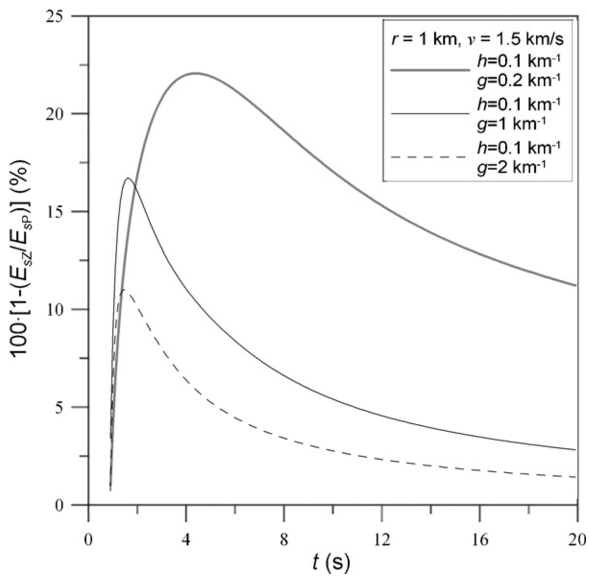

Fig. 2. Differences (in %) of the scattered wave energy estimations using the approximation of Paasschens (1997) (E_{sP}) and the hybrid single-scattering-diffusion solution by Zeng (1991) (E_{sZ}) as a function of time for typical attenuation parameters and characteristics of Mt. Vesusius region (Del Pezzo et al., 2006).

the difference between the exact and approximate solution is barely visible on this scale and it is of the order of 3% as a maximum in this example. Paasschens (1997) reported a difference of the order of 2% outside the ballistic peak and its tail. Fig. 1(b) shows the comparison for the case of shorter hypocentral distances (r = 10 km) and smaller S-wave velocities ($v = 2.5$ km/s). In this case, the difference of hybrid single-scattering-diffusion with the exact solution is of the order of 20%. As for the Paasschens (1997) expression, the difference between the exact and approximate solution is again of the order of 3% as a maximum.

In order to make a judgment on the accuracy of the hybrid single-scattering-diffusion approximate model in Vesuvius volcano region, we represent in Fig. 2 the errors in the synthetic scattered energy computations in accordance with the parameters reported by Del Pezzo et al. (2006). Thus, we consider very short hypocentral distances (r = 1 km) and an S-wave velocity value $v = 1.5$ km/s. Because the authors find a very strong predominance of scattering over intrinsic dissipation we show the scattered energy for the case of $h = 0.1\ \text{km}^{-1}$ and $g = 0.2$, 1, and $2\ \text{km}^{-1}$. Fig. 2 shows that, in general, the differences between the synthetic energy curves decrease with increasing scattering coefficient. For the case of $g = 2\ \text{km}^{-1}$ it is about 11% but it increases up to 22% for $g = 0.2\ \text{km}^{-1}$. Notice also that the shape of the envelopes also changes, since the error depends on time (see Fig. 2), thus strongly biasing the computation for both g and h.Therefore, we conclude that the hybrid single-scattering-diffusion solution may not be an accurate approximation for practical analysis in a wide range of lapse times and attenuation parameters. In fact, the solution by Zeng (1991) was reported by the author itself as a good approximation for the case of source and receiver co-located. However, Del Pezzo et al. (2006) use it in the general case without caring about its validity for arbitrary hypocentral distances and without checking the change in the shape of synthetic coda envelopes.

Nevertheless, there are several advantages in using an algebraic expression for synthesizing the multiple scattered wave envelopes. A very important one is the computation time, which is considerably less than for the two-dimensional integration process involved in the numerical calculation of the exact solution. Then, if an approximate solution is implemented in real applications, we believe that it is better to use the expression developed by Paasschens (1997) which is in all cases more accurate.

3. Point 2: stacked CODA envelopes

Del Pezzo et al. (2006) justify the stacking of coda envelopes by assuming that the available seismic events have similar coda envelopes for Mt. Vesuvius region. This assumption is based, on the one hand, on the observation that the coda waves for the seismic events considered share a common decay rate at each station. On the other hand, the earthquake sources considered are concentrated in a small volume, so the source-station paths share almost the same volume for a given source-station pair.

Therefore, in the work of Del Pezzo et al. (2006), it is assumed that the parameters g and h may be computed by fitting an average (stacked) S-coda envelope to a single synthetic curve where an average hypocentral distance is used as a fixed parameter. Nevertheless, there is neither theoretical nor numerical evidence that the shape of this average S-coda envelope may correspond to a single synthetic curve where the average hypocentral distance is used as a parameter and where g and h (determined numerically) correspond to the average of absorption and scattering coefficients of the region. Then, this method may give (unknown) biased computations of both g and h. We note here that in Eqs. (5) and (6) the dependence on the hypocentral distance is clear and it is a very important parameter, especially for the computation of g, since there is a product of g with a function of r^2 as an argument of an exponential function.

We also note that in the method used by Del Pezzo et al. (2006), the envelopes are only considered from two times the S-wave travel time (t_S). However, it is very well known empirically that, for local earthquakes recorded at times $t > 2t_S$, the envelope of a bandpass-filtered seismogram has a common shape that is independent of the source-receiver distance (Sato and Fehler, 1998) and its logarithm shows mainly a linear behavior (Rautian and Khalturin, 1978). Then, we wonder how it is possible to extract information of both g and h from a straight line. As we have already discussed in Point 1, the calculation of g and h is extremely sensitive to the shape of the envelopes. Therefore, we believe the stacking technique used by Del Pezzo et al. (2006) may provide biased estimations of the attenuation parameters in Vesuvius volcano region.

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References

Abubakirov, I.R., 2005. Attenuation characteristics of transverse waves in the lithosphere of Kamchatka estimated from observations at the Petropavlovsk digital broadband station, Izvestiya. Phys. Solid Earth 41, 813–824.

Bianco, F., Del Pezzo, E., Castellano, M., Ibáñez, J., Di Luccio, F., 2002. Separation of intrinsic and scattering seismic attenuation in the southern Apenninne zone, Italy. Geophys. J. Int. 150, 10–22.

Bianco, F., Del Pezzo, E., Malagnini, L., Di Luccio, F., Akinci, A., 2005. Separation of depth-dependent intrinsic and scattering seismic attenuation in the northeastern sector of the Italian peninsula. Geophys. J. Int. 161, 130–142.

Bouchon, M., 1979. Discrete wave number representation of elastic wave fields in three-space dimensions. J. Geophys. Res. 84, 3609–3614.

Cools, R., Haegemans, A., 2003. Algorithm 824: CUBPACK: a package for automatic cubature; framework description. ACM Trans. Math. Software 29 (3), 287–296.

Del Pezzo, E., Bianco, F., Zaccarelli, L., 2006. Separation of Q_i and Q_s from passive data at Mt. Vesuvius: a reappraisal of the seismic attenuation estimates. Phys. Earth Planet. Int. 159, 202–212.

Fehler, M., Hoshiba, M., Sato, H., Obara, K., 1992. Separation of scattering and intrinsic attenuation for the Kanto-Tokai region, Japan, using measurements of S-wave energy vs. hypocentral distance. Geophys. J. Int. 108, 787–800.

Goutbeek, F.H., Dost, B., van Eck, T., 2004. Intrinsic absorption and scattering attenuation in the southern part of the Netherlands. J. Seismol. 8, 11–23.

Hoshiba, M., Sato, H., Fehler, M., 1991. Numerical basis of the separation of scattering and intrinsic absorption from full seismogram envelope: a Monte–Carlo simulation of multiple isotropic scattering. Pap. Meteorol. Geophys. 42, 65–91.

Ishimaru, A., 1978. Wave Propagation and Scattering in Random Media. Academic Press, New York.

Matsunami, K., Nakamura, M., 2004. Seismic attenuation in a nonvolcanic swarm region beneath Wakayama, southwest Japan, J. Geophys. Res., 109, B09302, doi:10.1029/2003JB002758.

Paasschens, J.C.J., 1997. Solution of the time-dependent Boltzmann equation. Phys. Rev. E 56, 1135–1141.

Rautian, T.G., Khalturin, V.I., 1978. The use of coda for determination of the earthquake source spectrum. Bull. Seism. Soc. Am. 68, 923–948.

Sato, H., Fehler, M.C., 1998. Seismic Wave Propagation and Scattering in the Heterogeneous Earth, AIP Series in Modern Acoustics and Signal Processing. Springer–Verlag, New York.

Sens-Schönfelder, C., Wegler, U., 2006. Radiative transfer theory for estimation of the seismic moment. Geophys. J. Int. 167, 1363–1372.

Zeng, Y., 1991. Compact solutions for multiple scattered wave energy in time domain. Bull. Seism. Soc. Am. 81, 1022–1029.

Zeng, Y., Su, F., Aki, K., 1991. Scattered wave energy propagation in a random isotropic scattering medium. I: theory. J. Geophys. Res. 96, 607–619.